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A CLOSED FORM EXPRESSION FOR LINE-OF-SIGHT PROBABILITY

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U. S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY

ABERDEEN PROVING GROUND, MARYLAND

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This report examines expressions for line-of-sight probability as a function of observer altitude H and observer target range R. The result for a somewhat rough terrain classification is

$$P_{LOS} = 1 - \exp -(97.59267)H^{(0.689606)}/R$$

and the result for a smooth table-top terrain classification is

$$P_{LOS} = 1 - \exp -(770.748)H^{(0.6092)}/R.$$

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ACKNOWLEDGEMENT

A Closed Form Expression for Line-of-Sight Probability

PURPOSE

This report describes a closed form expression for the probability of line-of-sight (P_{LOS}) for a given terrain type as a function of observer altitude ! and observer-to-target range $\underline{R}$.

Figures illustrate the goodness-of-fit of "P_{LOS} vs R" curves to actual terrain and terrain classification type data.

A Closed Form Expression for Line-of-Sight Probability

1. BACKGROUND

For several years, those involved with modeling of line-of-sight probability (P_{LOS}) have been investigating analytic approximations for specific and classification type terrains. The authors propose the expressions in this text as a meaningful first step toward the accomplishment of establishing such analytic expressions.

2. ASSUMPTIONS AND HYPOTHESES

It is assumed that P_{LOS} is 1.00 when an observer looks down where looking down defines a zero elevation angle, $\theta = 0$ degrees. The expression selected should yield "P_{LOS} vs. R (observer/target range)" curves that exhibit a slight rounding at the near range. That is, P_{LOS} is approximately 1.00 for elevation angles less than 60 degrees (Ref. 1). An initial hypothesis is formed by taking the exponential of a cotangent (ctn) function on the elevation angle

$$P_{LOS} = 1 - \exp\,(-a\,\text{ctn}\,\theta) \qquad\qquad (1)$$

where elevation angle $\underline{\theta}$ is interpreted as the arctangent of the ratio of range and observer attitude.

$$\theta = \text{Arctan}\,(R/H) \qquad\qquad (2)$$

The condition that $0 \leq P_{LOS} \leq 1$ requires that "$\underline{a}$" be positive.

3. DEVELOPMENT OF P_{LOS} EXPRESSION

An "$\underline{a}$" family of "P_{LOS} versus θ" curves is shown in Figure 1. The "$\underline{a}$" parameter characterizes a rate at which line-of-sight is lost. This rate roughness factor reflects the combined degradation effects of terrain curvature, target height/profile, and vegetation.

Combining equations 1 and 2 yields,

$$P_{LOS} = 1 - \exp\,(-a\,H/R) \qquad\qquad (3)$$

Figures 2 and 3 illustrate equation 3 as an "H" family of "P_{LOS} vs. R" curves. Figure 2 applies for a rate roughness value, $a = 10$ while Figure 3 applies for a rate roughness value, $a = 20$. Maintaining a constant product of a and H will maintain the same P_{LOS} value. For example, if $\underline{a}$ is doubled and H is halved, P_{LOS} remains unchanged for a given range.

Figure 4 illustrates the behavior of a factor $\underline{b}$ which when multiplied by elevation angle θ increases the P_{LOS} asymptote as $\underline{R}$ approaches infinity. The expression plotted in Figure 4 is

$$P_{LOS} = 1 - \exp\left(-a\, \text{ctn}\, (b\, \arctan\, (R/H))\right). \tag{4}$$

Since $\underline{b}$ affects the asymptotic value of the "P_{LOS} vs. R" curve, it is referred to as the asymptote roughness factor. The need for considering non-unity values ($0<b<1$) for this asymptote factor was hypothesized as necessary since real P_{LOS} data appear to show curves that approach values greater than zero at far ranges (Ref. 3).

Equation 4 was fitted to three specific maps of the 5Dd classification for various observer altitudes (Ref. 2).* Figures 5 through 10 illustrate that equation 4 fits P_{LOS} data well for each specific map. However, these same figures demonstrate the variation in rate roughness factor $\underline{a}$ with observer altitude $\underline{H}$. This variation of $\underline{a}$ versus $\underline{H}$ challenges the hypothesis that P_{LOS} is fundamentally a function of elevation angle θ. That is, P_{LOS} remains constant for a given ratio of R and H. Therefore, with this allowance of a false hypothesis on θ, $\underline{a}$ is fitted with $\underline{H}$. In addition, Figures 5 through 10 illustrate that parameter $\underline{b}$ is constant at 1.00.**

To characterize the terrain roughness for the 5Dd terrain classification, three terrain samples in the 5Dd classification were analyzed.***

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*This classification is detailed in Table 1 as extracted from Ref. 2. In a descriptive sense, 5Dd is rough terrain.
**For 19 of 21 terrain fits (one fit for each of seven observer altitudes $\underline{H}$ for each of three terrains), the $\underline{b}$ value was 1.00. On the other two cases, the $\underline{b}$ value was 0.99.
***Preselection process of what terrains were to be analyzed was based on the availability of tapes with digitized terrain characteristics and the knowledge that the terrains selected were different from each other by geographic location and general appearance. The Appendix gives a listing of terrain maps analyzed.

The $\underline{a}$ values were averaged over these three terrains at each observer altitude H. Then these $\underline{a}$ values were least squares fitted as a function of observer altitude $\underline{H}$. Equation 5 is the result of this fitting process.

$$a = (97.59267) \, H^{(-0.310394)} \tag{5}$$

The resulting P_{LOS} expression for the 5Dd terrain classification is*

$$P_{LOS} = 1 - \exp\left(-(97.59267) \, H^{(0.689606)}/R\right). \tag{6}$$

Figures 11 through 17 illustrate the fit of equation 6 to the "P_{LOS} vs. R" curves of the three 5Dd terrains from which parameters for equation 6 were derived.

4. RESULTS

The results from Figure 1 are:

a. Larger values of $\underline{a}$ imply that the probability of line-of-sight is greater.

b. The rate roughness factor $\underline{a}$ causes marked changes in curves of P_{LOS} vs. elevation angle when given values between 0.01 and 10.

Results from Figure 5 through 10

a. For specific terrain and observer height, equation 3 does fit data of "P_{LOS} vs. R" well.

b. Equation 3 is not sufficient to depict the P_{LOS} characteristics as both a function of range and observer height. To obtain such an expression involves fitting the dependence of the rate roughness factor $\underline{a}$ on observer altitude $\underline{H}$. (See Appendix)

$$a = AH^B$$

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*The Appendix provides a description of the programs involved to conduct a fit of equation 4 to digitized terrain data. Also provided is the subsequent analysi to arrive at this expression.

Results from Figures 11 through 17 are:

a. Variations in "P_{LOS} vs. R" curves among terrains in the same classification appear large.

b. Equation 6 does yield a "P_{LOS} vs. R" curve within the variation of the three terrains of classification 5Dd from which the two parameters in equation 6 were obtained.

5. RECOMMENDATIONS

a. A similar analysis/fit should be conducted on all of the Natick classifications to form a data base for immediate use by expected value and/or Monte Carlo simulation/wargame models.

b. The Natick classification scheme could be improved by grouping terrains into categories determined by a specified variance about given "P_{LOS} vs. R" curves.

6. COMMENT

A similar analysis was conducted on three terrains of the Natick Classification 1Aa - a smooth, table-top terrain. Results/fits are as good as, if not better than, those shown in this report on the 5Dd classification. The corresponding equation 6 for this 1Aa classification is

$$P_{LOS} = 1 - \exp{(-770.748 \ H^{(0.6092)}/R\,)} \tag{6a}$$

REFERENCES

1. Burge, Carol J., "A Revision of Surface-to-Surface Masking Studies,"
June 1975, NWC-TP-5773.

2. Anstey, COL R. (QM USAR), "The Natick Landform Classification System,"
US Army Materiel Systems Analysis Activity, TR100.

3. Olson, Warren K., "A Terrain Analysis of Four Tactical Situations,"
TM158, Dec 1972, AD90934.

4. Defense Mapping Agency, "Catalogue of Maps, Charts, and Related Products"
Map Series M745.

TABLE I. CLASS INTERVALS OF CLASSIFICATION DESCRIPTORS

Maximum Hill Height (Local Relief)

Descriptor	Class Interval	
1	0 -10 Meters	0 - 33 Feet
2	10 -30	33 - 99
3	30 -50	99 - 165
4	50 -100	165 - 330
5	100 -300	330 - 990
6	OVER 300	OVER 990

Modal Hill Height (Local Relief)

Descriptor	Class Interval	
A	0 -10 Meters	0 - 33 Feet
B	10 -20	33 - 66
C	20 -35	66 - 115
D	35 -50	115 - 165
E	50 -75	165 - 248
F	75 -100	248 - 330
G	100 -125	330 - 413
H	125 -150	413 - 495
I	150 -175	495 - 578
J	175 -200	578 - 660
K	OVER 200	OVER 660

Number of Positive Features Per Mile

Descriptor	Class Interval	
a	0 -0.8/Kilometer	0 - 0.5/Mile
b	0.8 -1.6	0.5 - 1.0
c	1.6 -2.4	1.0 - 1.5
d	2.4 -3.2	1.5 - 2.0
e	3.2 -4.0	2.0 - 2.5
f	OVER 4.0	OVER 2.5

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APPENDIX

A.1. PROCESS TO OBTAIN P_{LOS} EXPRESSION

Obtaining a closed form expression for P_{LOS} involves three steps:

<u>Step 1:</u> Generating P_{LOS} curves from digitized terrain data and fitting equation 4 to these curves,

<u>Step 2:</u> Obtaining a correlation function between rate roughness factor <u>a</u> and observer altitude <u>H</u>, and

<u>Step 3:</u> Calculating/plotting of the theoretical/fitted curve.

Step 1 consists of the program "real curve fit." This program is a combination of two programs. The primary program "LOS RIngs," written by Warren Olsen, generates the "P_{LOS} vs. R" curves from digitized terrain data (Ref. 3).* The secondary program "Minmax" modeled by J. Sheldon and written/ executed by Sean Smith, fits equation 4 to the digitally processed "P_{LOS} vs. P" curve by using the Kolmogorov-Smirnov, maximum difference test for Goodness-of-Fit.

The program "Real Curve Fit" is written in FORTRAN IV compatible only with the BRLESC II computer at Aberdeen Proving Ground, Maryland. Table II illustrates the information necessary to run a single case - a case consists of a "P_{LOS} vs. R" fit for each of seven observer altitudes.

Table II

Computational Information on BRLESC II Program - Real Curve Fit

	Range KM		
	4	6	8
Run Time (Min)	45	71	105
Output (Lines)	600	735	870

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*The P_{LOS} result from digitized terrain is an average result. The averaging occurs over observer location on the terrain over a 360 degree scan. This process accounts for the somewhat smooth curve representing the actual "P_{LOS} vs. R" result.

Step 2, finding an equation which relates rate roughness factor a to observer altitude H, uses a standard/library program "Regression Analysis with Plotting." This process begins by averaging all the rate roughness factors a over the three terrains for each of the observer heights H. These average values of a are least squares fitted to H in the form*

$$a = AH^B. \tag{7}$$

Step 3 consists of the plotting of equation 6 which is the result of combining equations 3 and 5. The general form for equation 6 is

$$P_{LOS} = 1 - \exp\left(-AH^{(B+1)}/R\right) \tag{8}$$

A.2. TABLE OF ACTUAL TERRAINS STUDIED

Table III lists the terrains studied in this report. These terrains are shown in Figure A.1.

TABLE III

Actual Terrains Studied

Natick Terrain Classification	DMA MAP Numbers	Actual Terrain Code**/City
5Dd	L6134 - L6132 L5120 - L5118 L4920 - L4918	G-93/Bayreuth G-48/Stadt Allendorf G-44/Bad Wildungen
1Aa	L7130 - L7128 L7332 - L7330 L7334 - L7332	G-70/Nordlingen" G-76/Neuberg G-77/Ingolstadt

*The correlation function between a and H was tested in two different forms ($a=AH^B$ and $a=AB^H$). The best results were obtained by using the former expression, Equation 7.
**Terrain codes are standard DMA (Defense Mapping Agency) codes (Ref. 4).
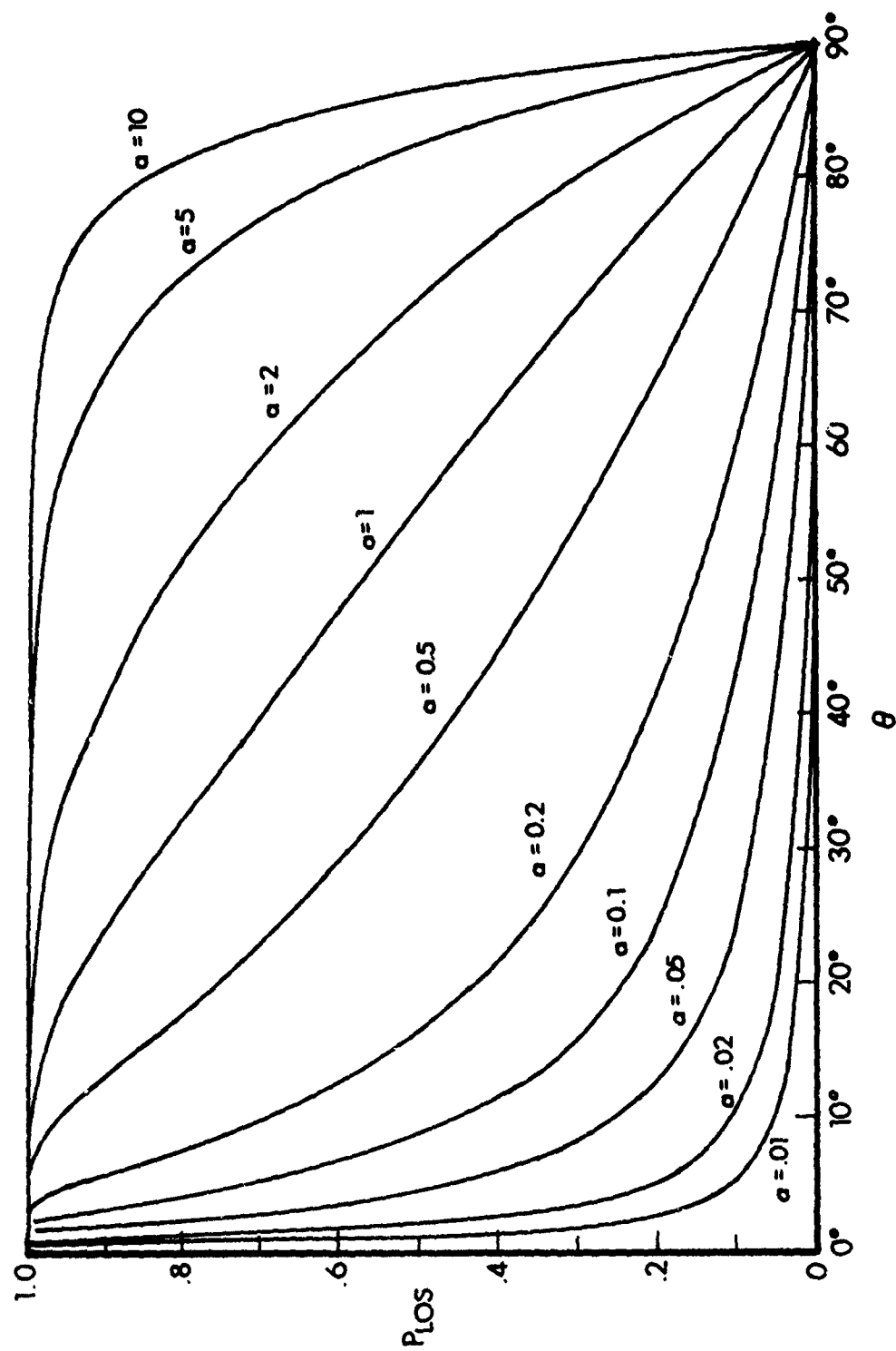
Figure 1. An "a" Family of "P_{LOS} vs θ" Curves.

$$P_{LOS} = 1 - \exp(-a\,CTN\,\theta)$$
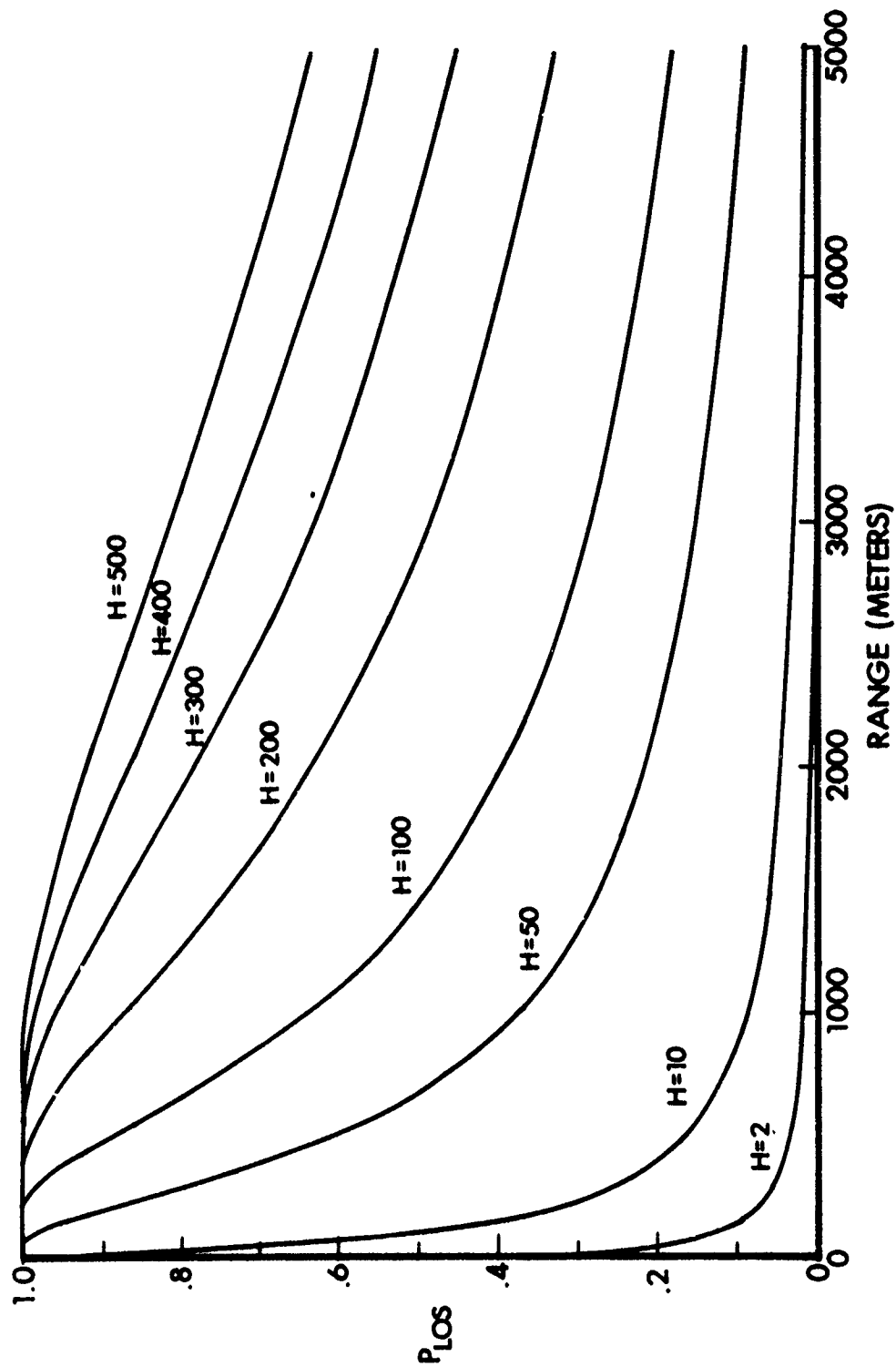
Figure 2. An "H" Family of "P_{LOS} vs R" Curves (a=10)

$$P_{LOS} = 1 - exp\ (-a\ H/R)$$

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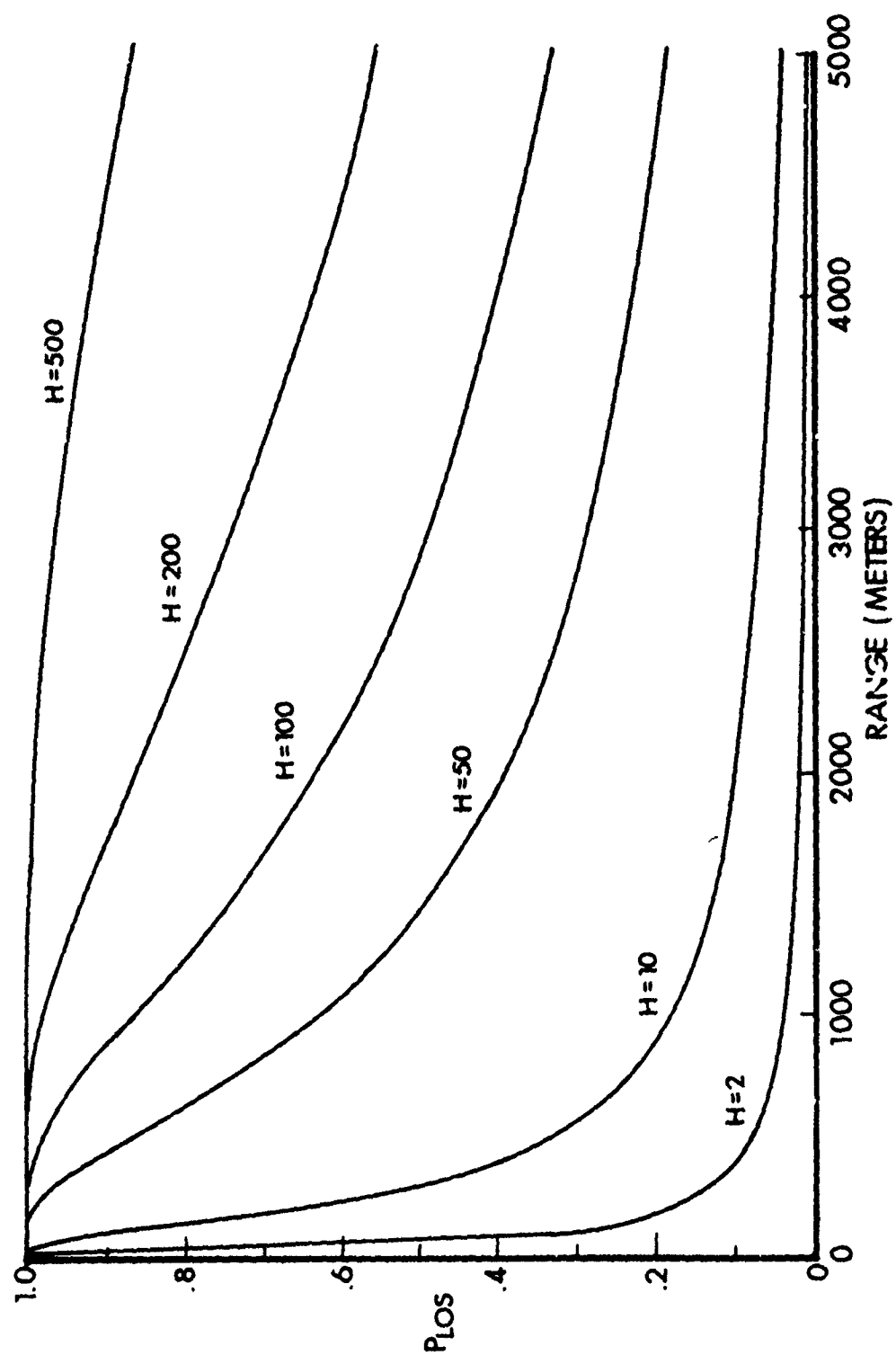
Figure 3. An "H" Family of "P_{LOS} vs R" Curves (a=20)
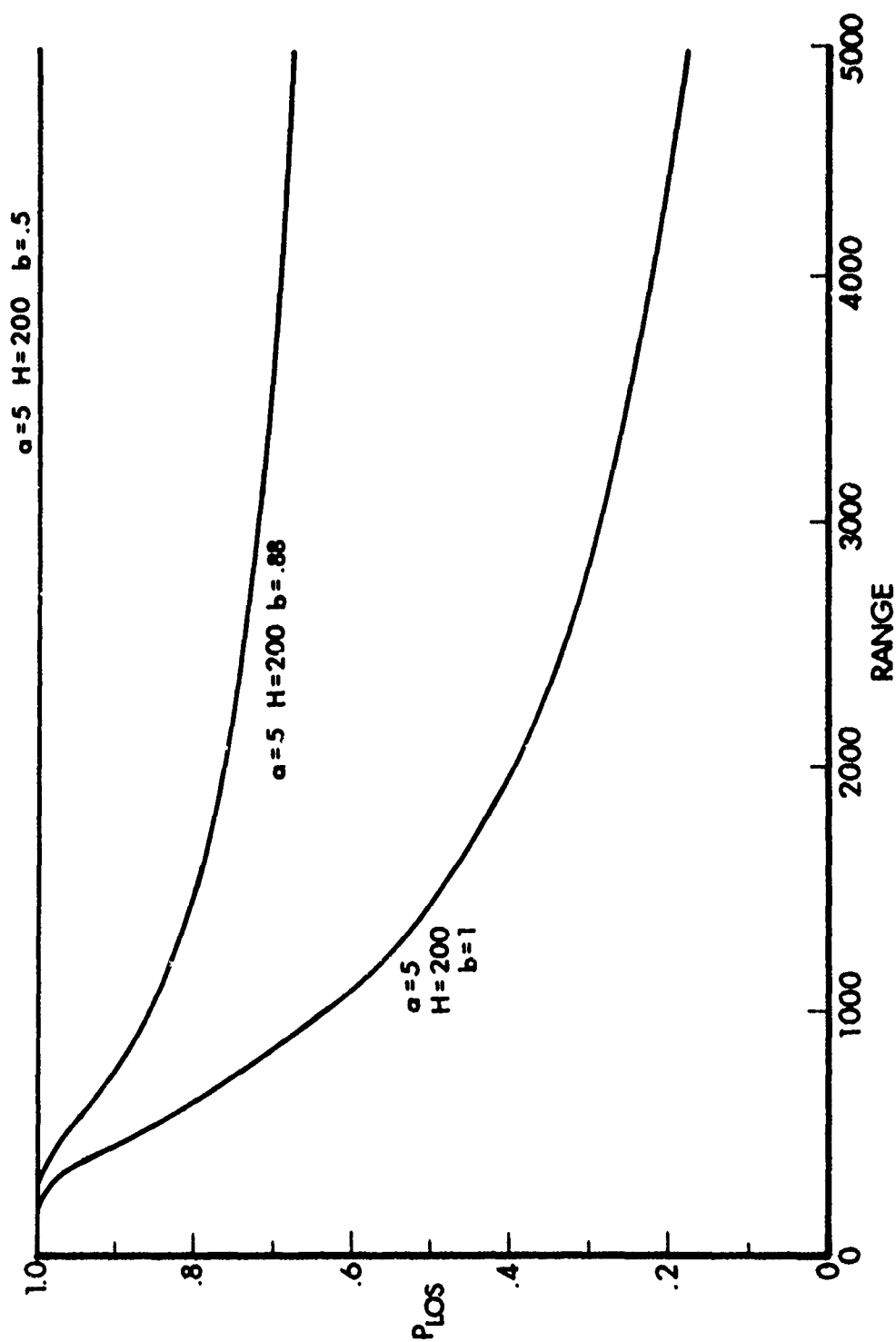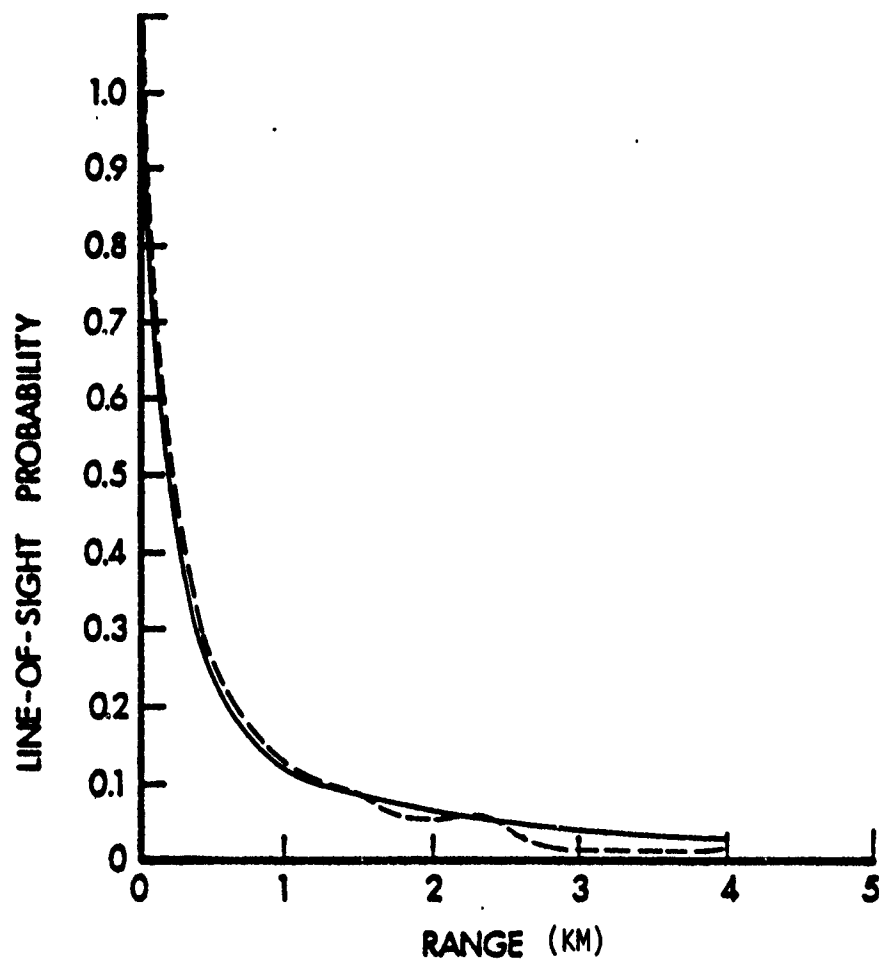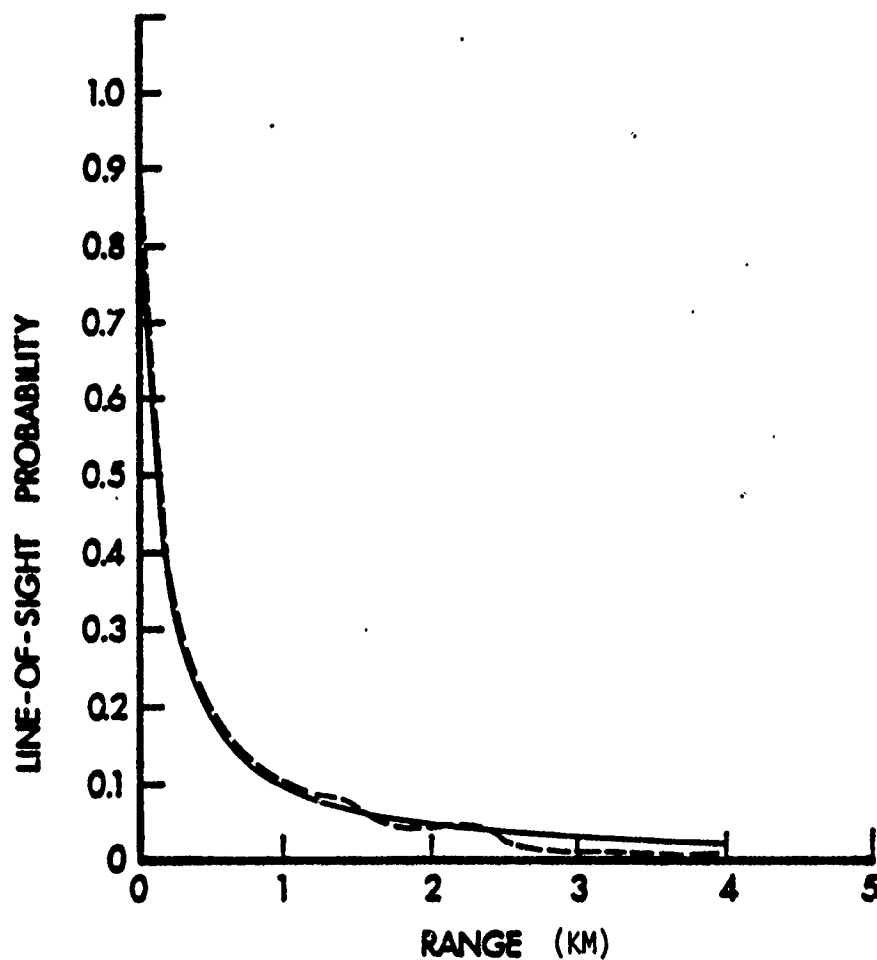
$$P_{LOS} = 1 - \exp(-a\,H/R)$$

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Figure 4. A "b" Family of P_{LOS} vs R.

$$P_{LOS} = 1 - \exp\left(-a \ \text{ctn} \left(b \ \text{artan} \left(R/H\right)\right)\right)$$

OBSERVER HEIGHT = 3.00 M
TARGET HEIGHT = .00
AREA G93
CLASSIFICATION 5D d
RATE SHAPE FACTOR a = 43.11
ASYMPTOTIC SHAPE FACTOR B = 1.00
MINIMUM MAXIMUM DIFFERENCE = 0.03

Figure 5   Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)
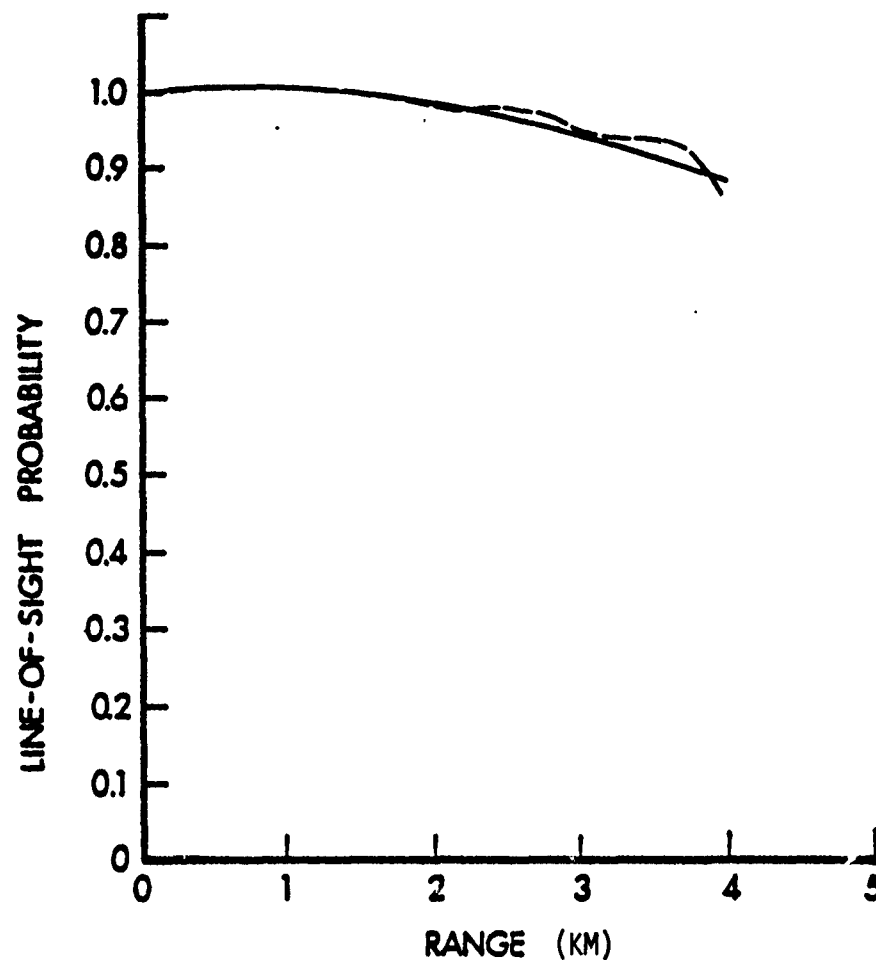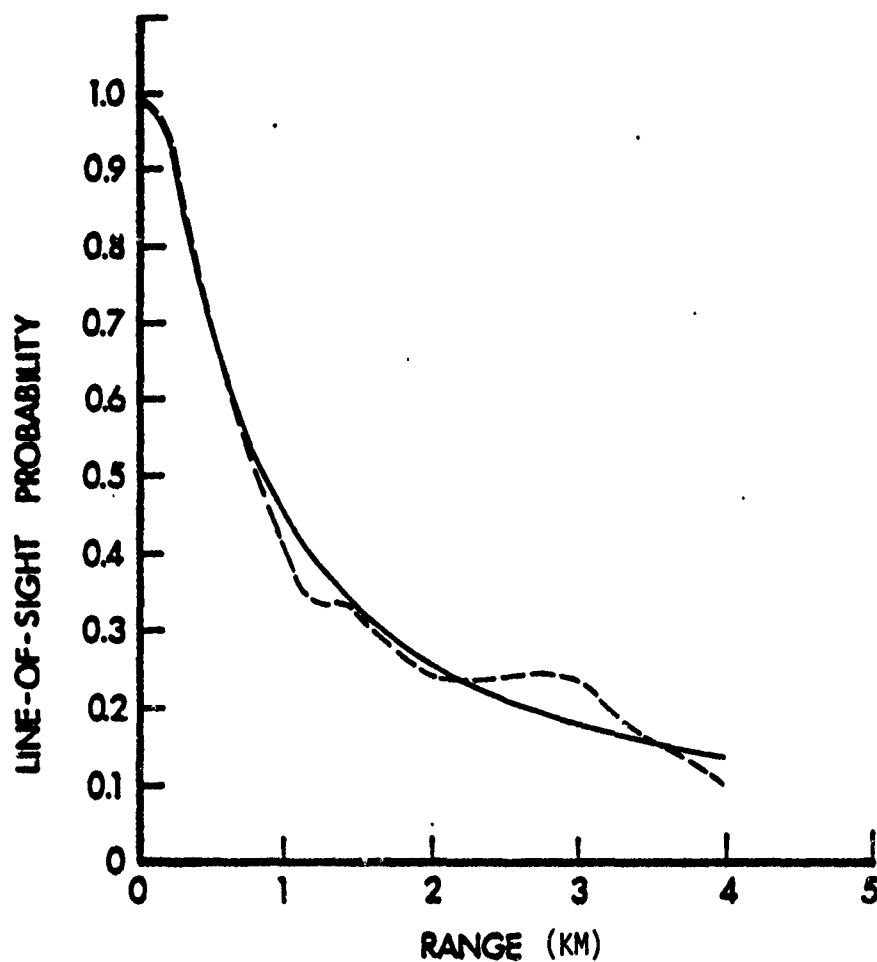
OBSERVER HEIGHT = 1.67 M
TARGET HEIGHT = .00
AREA G93
CLASSIFICATION 5Dd
RATE SHAPE FACTOR a = 61.96
ASYMPTOTIC SHAPE FACTOR B = 1.00
MINIMUM MAXIMUM DIFFERENCE = 0.02

Figure 6   Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)
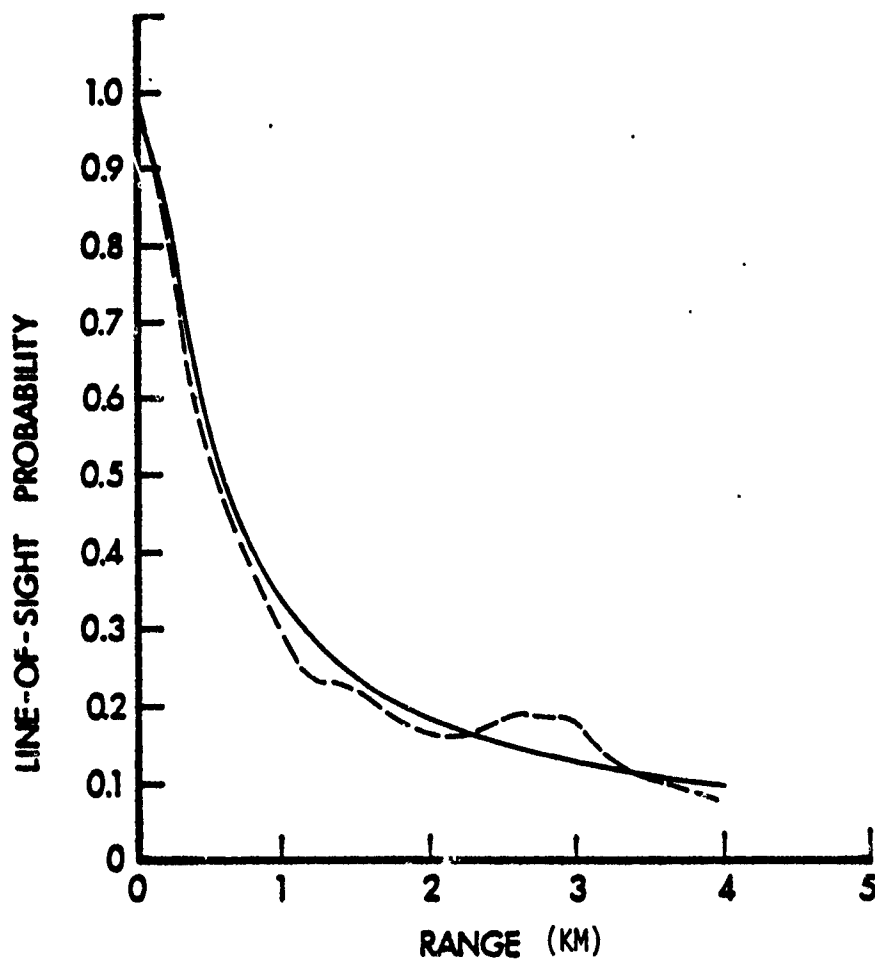
OBSERVER HEIGHT = 10.00 M
TARGET HEIGHT = .00
AREA G44
CLASSIFICATION 5D d
RATE SHAPE FACTOR a = 40.76
ASYMPTOTIC SHAPE FACTOR B = 1.00
MINIMUM MAXIMUM DIFFERENCE = 0.05

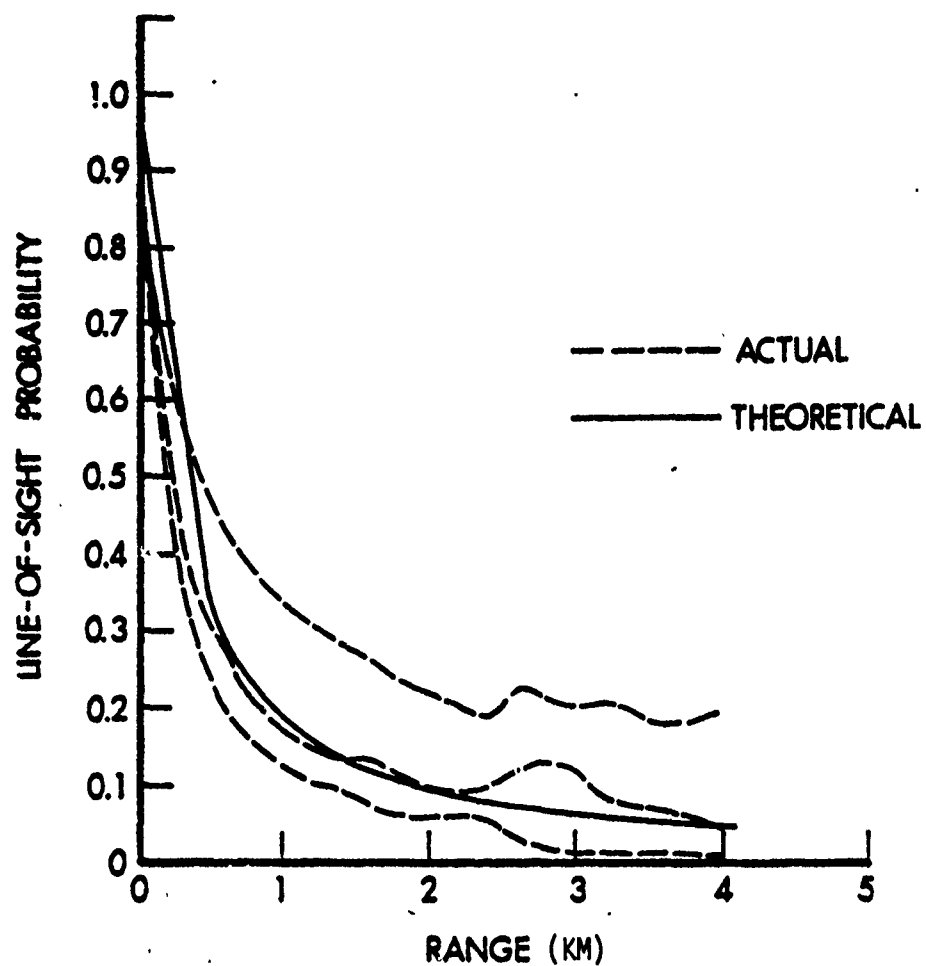Figure 7   Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)
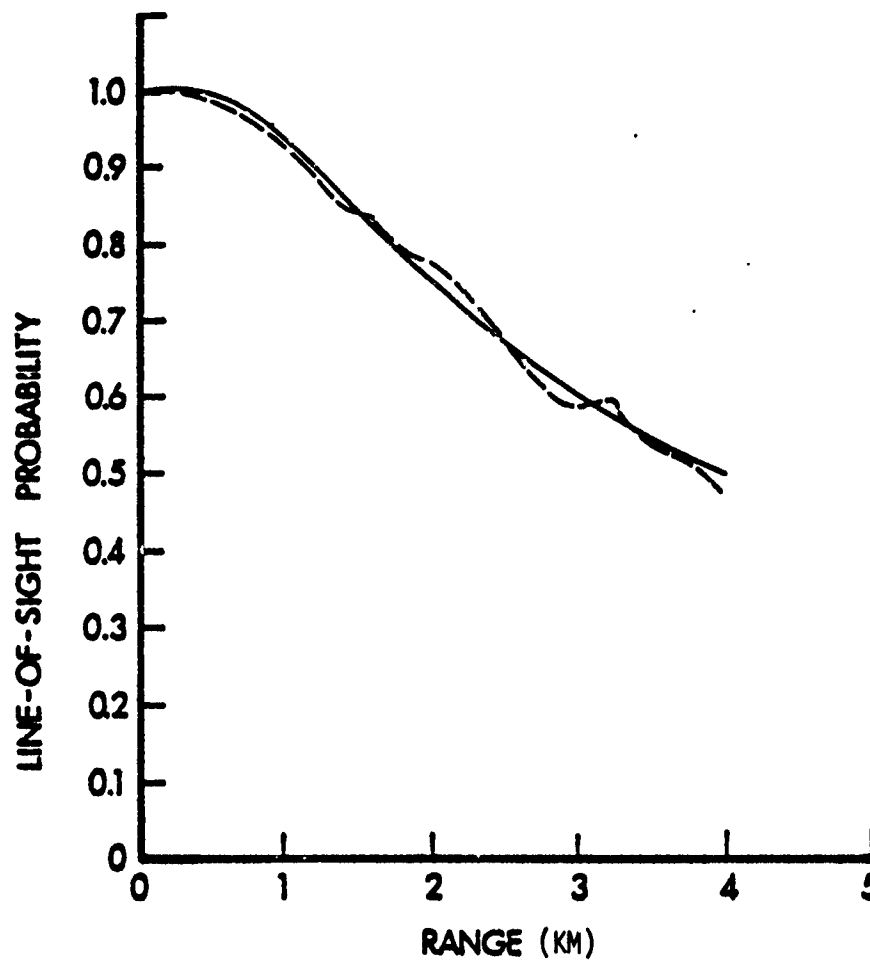
OBSERVER HEIGHT = 100.00M
TARGET HEIGHT = .00
AREA G48
CLASSIFICATION 5Dd
RATE SHAPE FACTOR a = 27.67
ASYMPTOTIC SHAPE FACTOR B = 1.00
MINIMUM MAXIMUM DIFFERENCE = 0.03

Figure 8 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)

OBSERVER HEIGHT = 500.00M
TARGET HEIGHT = .00
AREA G93
CLASSIFICATION 5Dd
RATE SHAPE FACTOR   a = 15.36
ASYMPTOTIC SHAPE FACTOR B = 0.99
MINIMUM MAXIMUM DIFFERENCE = 0.03

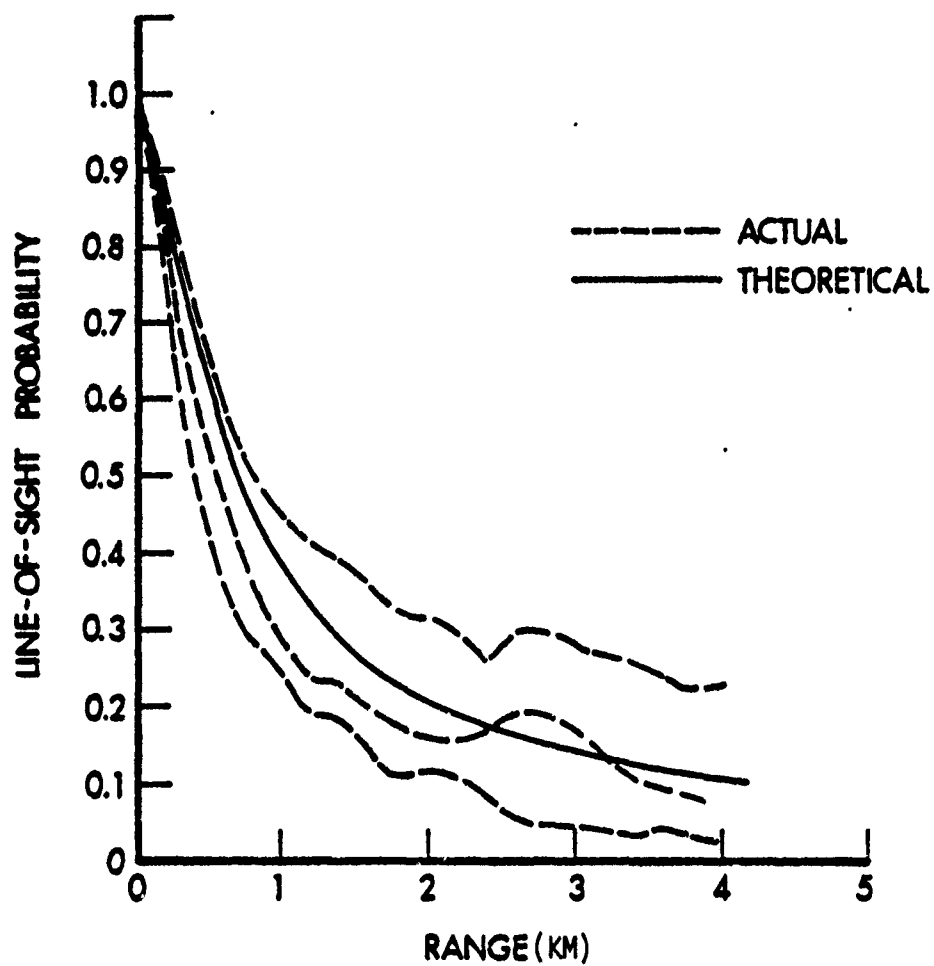Figure 9   Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)

OBSERVER HEIGHT = 20.00 M
TARGET HEIGHT = .00
AREA G44
CLASSIFICATION 5D d
RATE SHAPE FACTOR a = 29.70
ASYMPTOTIC SHAPE FACTOR B = 1.00
MINIMUM MAXIMUM DIFFERENCE = 0.06

Figure 10 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)

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Figure 11 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)
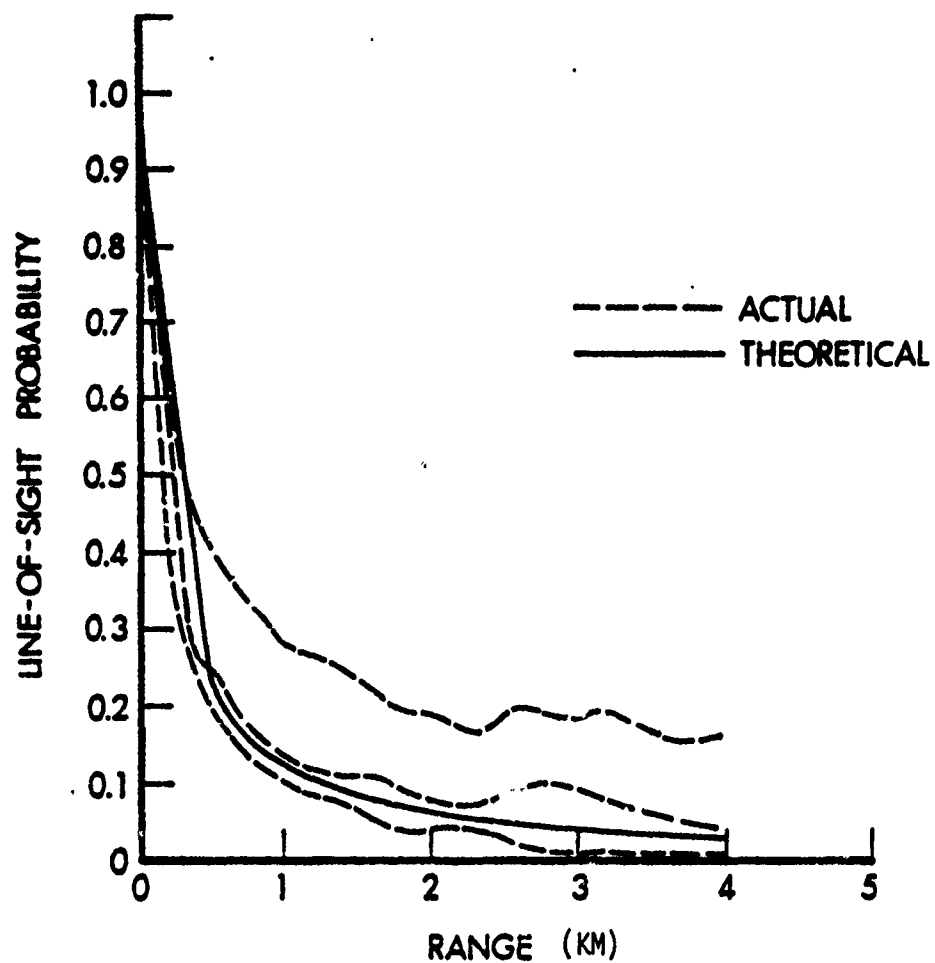
OBSERVER HEIGHT = 1.67 M
TARGET HEIGHT = .00
ROUGHNESS FACTOR a = 83.24
ROUGHNESS FACTOR B = 1.00
CLASSIFICATION 5Dᵈ

OBSERVER HEIGHT = 3.00 M
TARGET HEIGHT = .00
ROUGHNESS FACTOR a = 69.39
ROUGHNESS FACTOR B = 1.00
CLASSIFICATION 5Dᵈ

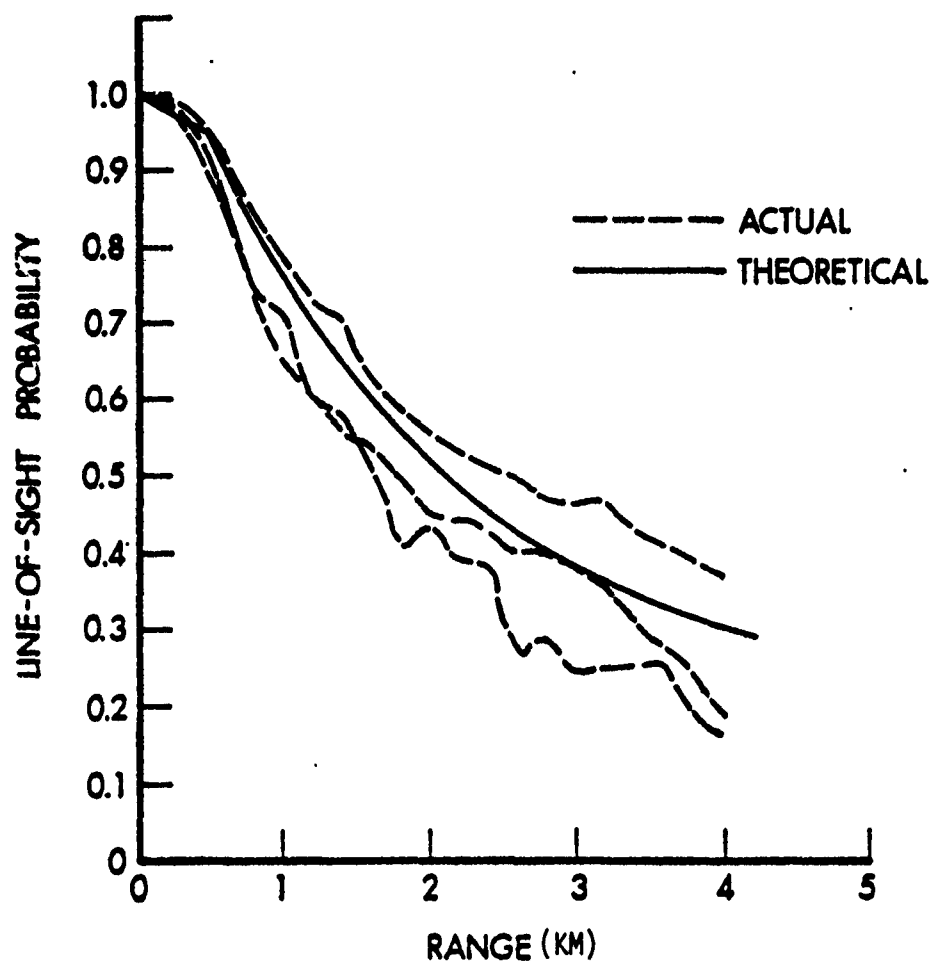Figure 12 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)

OBSERVER HEIGHT = 10.00 M
TARGET HEIGHT = .00
ROUGHNESS FACTOR a = 47.75
ROUGHNESS FACTOR B = 1.00
CLASSIFICATION 5D d

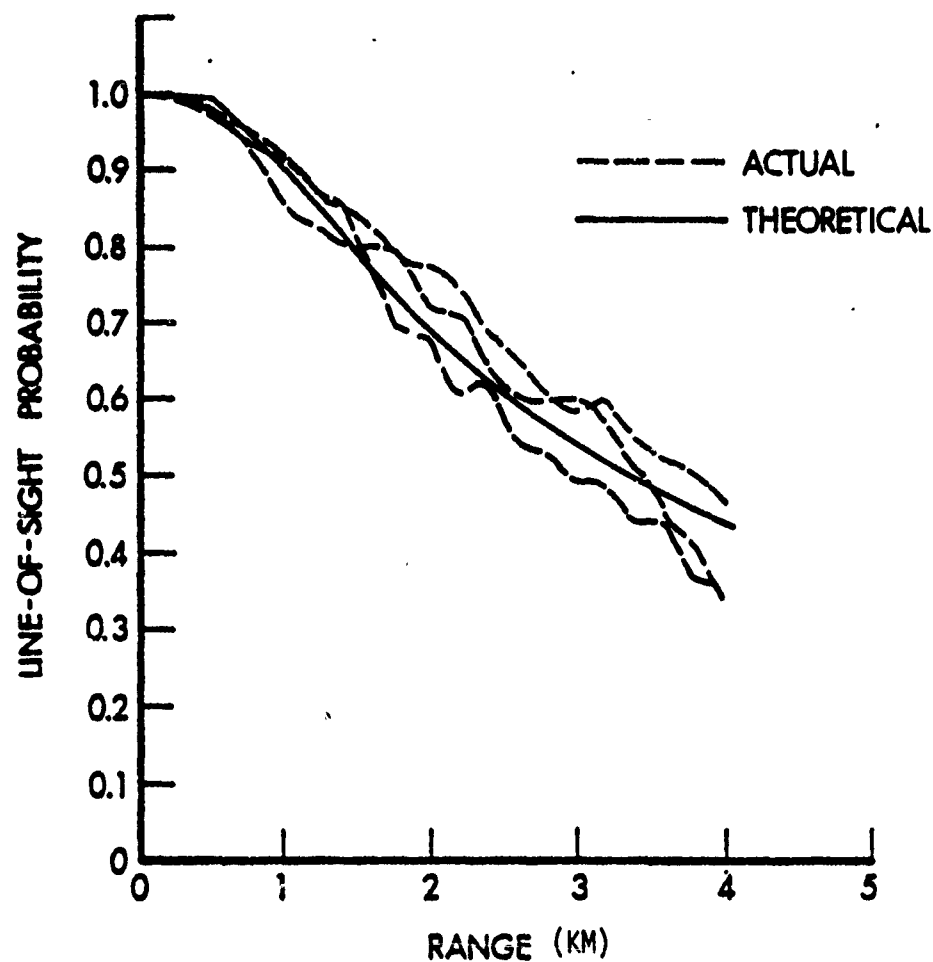Figure 13 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)
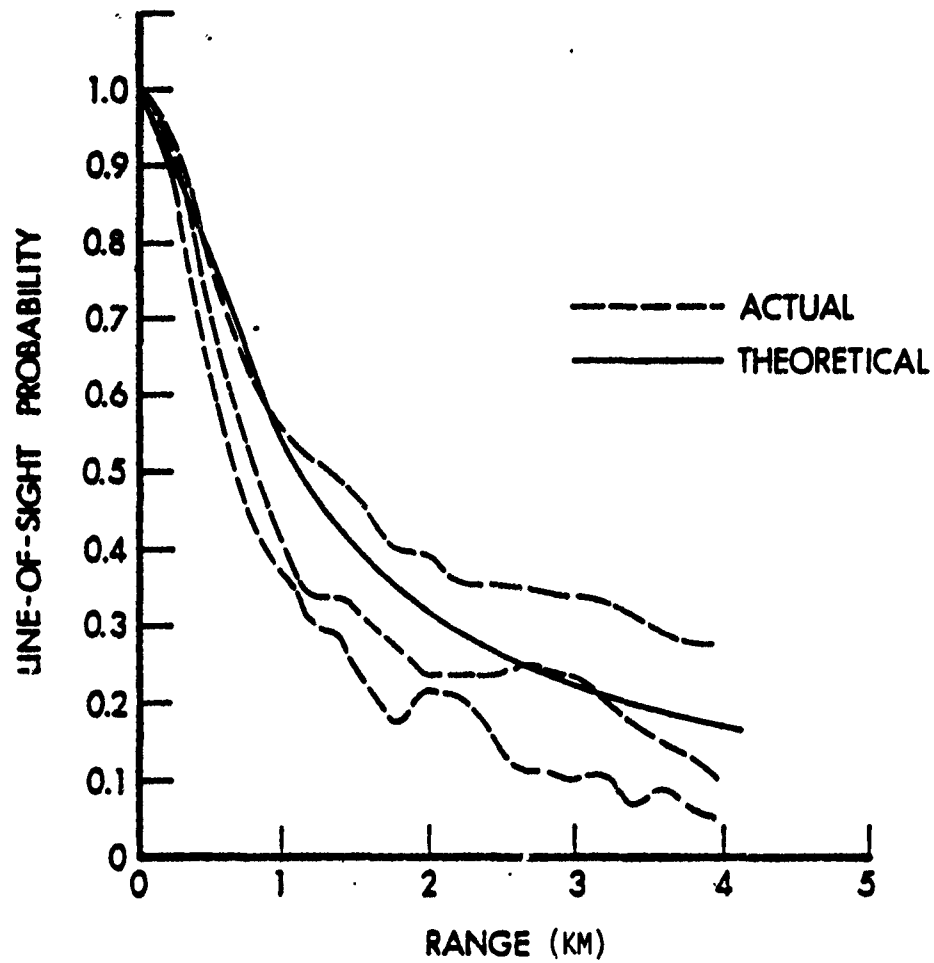
OBSERVER HEIGHT = 20.00 M
TARGET HEIGHT = .00
ROUGHNESS FACTOR a = 38.51
ROUGHNESS FACTOR B = 1.00
CLASSIFICATION 5Dd

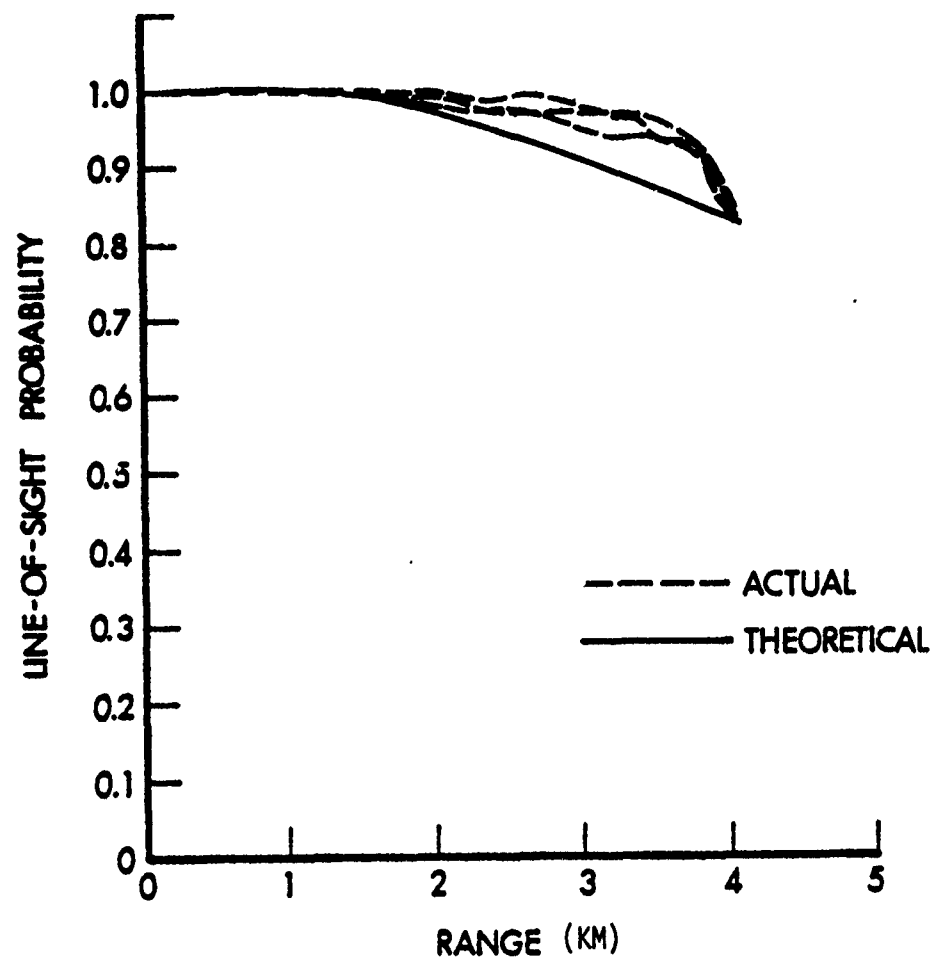Figure 14 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)

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OBSERVER HEIGHT = 50.00 M
TARGET HEIGHT = .00
ROUGHNESS FACTOR a = 28.98
ROUGHNESS FACTOR B = 1.00
CLASSIFICATION 5D d

Figure 15 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)

Figure 16 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)

OBSERVER HEIGHT = 500.00 M
TARGET HEIGHT = .00
ROUGHNESS FACTOR a = 14.18
ROUGHNESS FACTOR B = 1.00
CLASSIFICATION 5D d

Figure 17 Line-of-Sight Probability vs. Range.
(Artificial Curve Fit)

WEST GERMANY
SERIES MAP   M745

44 G-44 } ROUGH TERRAIN
48 G-48 }

ROUGH TERRAIN
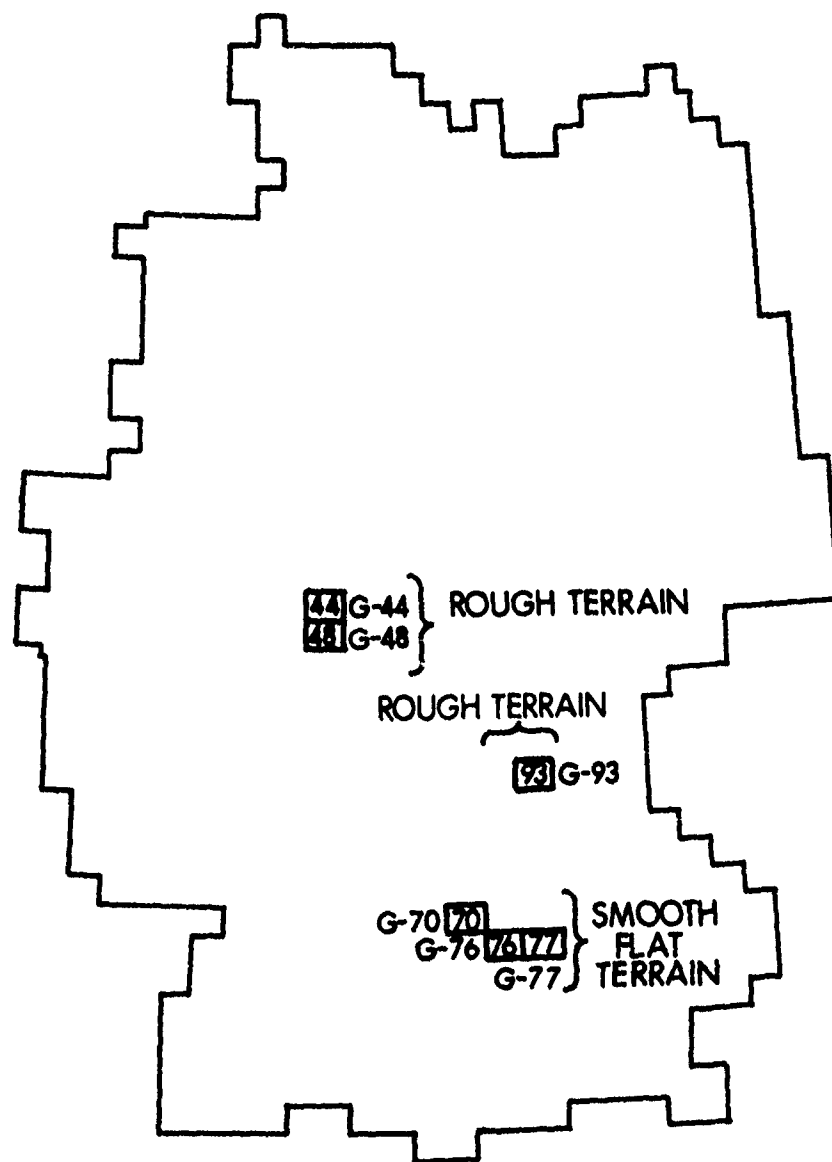
93 G-93

G-70 70
G-76 76 77 } SMOOTH
G-77   } FLAT
         } TERRAIN

Figure A-1. Location of Terrain Areas.